RUTGERS
THE STATE UNIVERSITY OF NEW JERSEY

Homework 2 (01:640:356 THEORY OF NUMBERS)

Spring 2023

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1. **Congruent Sum**

   Suppose that $p > 3$ is a prime number. Prove that

   $$2^{p-2} + 3^{p-2} + 6^{p-2} \equiv 1 \mod p$$

2. **Carmichael numbers fact**

   We studied that Carmichael numbers are those that are pseudoprime in every base for a given number n. Alternatively, it is known that m is a Carmichael number if $m = q_1 q_2 \cdots q_s$, where the q_i are distinct primes and $(q_i - 1)|(n-1)$ for every $i \in \{1, 2, \ldots, s\}$. Use this fact to prove that every Carmichael number is odd and has at least three different prime factors.

3. **Euler's phi**

   Find all y, such that $\phi(y) = 10$.

4. **Double sum**

   Find

   $$\sum_{d_1|n} \sum_{\substack{d_2|n,\\ d_3|n\\ (d_2,d_3)=d_1}} d_3$$

   as a function of n.

5. **Schedule**

   Set up a round-robin tournament schedule for 11 teams, i.e., create a schedule with 10 rounds, such that in each round, each team plays exactly one match and after the 10 rounds, every team has played every other team exactly once. Just describe how you would do it, without giving the complete schedule (only the first few rounds).

6. **Product**

   Show that if $n \in \mathbb{Z}^+$ and if $\mu(x)$ denotes the function of Möbius, then

   $$\prod_{j=0}^{j=3} \mu(n+j) = 0$$