Homework Problems (01:640:437 HISTORY OF MATHEMATICS)

Fall 2022

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1. **Ultramagic square**

   An *ultramagic* square is a magic square with the additional property that the sums in all *pseudo-diagonals* are also equal to the sum in each row, column and in each of the two proper diagonals. A pseudodiagonal is a diagonal consisted of two non-proper diagonals having n numbers in total and being on the opposite sides of a proper diagonal. For example, in the 4×4 square below, the numbers 10, 13, 7 and 4 are forming a pseudodiagonal since these are four numbers comprising two non-proper diagonals (respectively - 10, 13, 7 and 4).

1	12	7	14
8	13	2	11
10	3	16	5
15	6	9	4

   The other pseudodiagonals are formed by the numbers:
   $8, 12, 9, 5$
   $1, 6, 16, 11$
   $8, 3, 9, 14$
   $10, 6, 7, 11$
   $15, 12, 2, 5$.

   As you can see, the sum in each of these pseudodiagonals is 34, and this is the same as the sum in each row, each column and in each of the two proper diagonals ($1, 13, 16, 4$ and $15, 3, 2, 14$). Therefore, the given square is an ultramagic square. Prove that there is no 3×3 ultramagic square, except the trivial such square for which every entry has the same value?

2. **Egyptian fractions for** 1

   It is known that there is always a representation of 1 as an Egyptian fraction with distinct even denominators of any length greater than 3. Use that $\frac{1}{2D} = \frac{1}{2D+2} + \frac{1}{2D(D+1)}$, to find such representations for $4, 5$ and 6 distinct even denominators.

3. **Consecutive triples of Pythagoras**

   Prove that if we have 3 consecutive numbers a, b and c, which are the sides of a triangle with a right angle, then these numbers must be 3, 4 and 5.

4. **Sums of triangular numbers**

   Prove that every number greater than 1, which is a perfect square, is the sum of two consecutive triangular numbers, i.e., of the numbers of the kind $\frac{m(m+1)}{2}$, for some number m.

5. **Chinese Reminder Theorem**

   Find the smallest multiple of 10 which has remainder 3 when divided by 7, and remainder 2 when divided by 3.

6. **Quartic equation**

   Solve the quartic equation $3y^4 + 6y^3 - 57y^2 - 204y - 180 = 0$ using the method of Ferrari explained in class. Do not just give the roots (there are several calculators online), but describe each step in the process.

7. **Binomial theorem problem**

   Use the binomial theorem $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$ to simplify the expression

   $$\frac{(a+1)^4 + (a-1)^4}{(a+1)^4 - (a-1)^4}.$$

   Who is credited to have found the general version of the binomial theorem, where the exponent can be any rational number?