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W9: The development of Probability and Statistics.

November 7, 2022

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Aristotle believed there are 3 kinds of events:

1. certain events
2. probable events
3. unpredictable events (i.e., those occurring in games of chance)

However, numerical patterns in games of chance were not analysed until 1000 AD.

The unfinished game

Pascal and Fermat, play a simple game: A fair coin is tossed up to five times. If the outcome is Head, Pascal wins 1 point. Otherwise, Fermat wins 1 point. The game stops when any of the players win 3 points. Whoever wins take the price of 100 coins. Assume that the current result is 2 : 1 for Pascal and they need to stop playing the game. What is a fair way to distribute the 100 coins?

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Answer: 75 coins for Pascal and 25 for Fermat.

Without the concept of probability, this problem seemed to not be that elementary, as it is for us today.

In fact, this problem was known 150 years before Pascal and Fermat..

Some people we already discussed contemplated on the same question.

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(1447-1517)

Gerolamo Cardano
(1501-1576)

Niccolo Tartaglia
(1500-1557)

– Pacioli (a friend of Leonardo Da Vinci) first put the question in print, but with final goal - 6 points and score 5 : 2. He suggested (incorrectly) a division of 5 to 2 for the prize, according to the current score.

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– Cardano and Tartaglia corrected Pacioli, but did not get a solution.

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- Cardano wrote "The book on Games of Chance" in 1564. The book does not contain general theory, but instructions for gamblers.
- Tartaglia believed that the unfinished game problem is impossible to solve.

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– In his twenties, he got ill (he was probably gluten intolerant) and did not recover until his dead.

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The Pascal's triangle and the binomial theorem

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
    1   6   15  20  15  6   1
  1   7   21  35  35  21  7   1
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The famous "Pascal's wager" (part of *Pensees*):
If God does not exist, one will lose nothing by believing in him, while if he does exist, one will lose everything by not believing.

Pierre de Fermat (1601-1665)

– Born in Bordeaux, south France.

– Studied law and then mathematics. He did not have any position as math professor, but spent immense amount of time on math and was one of the brightest!

Most famous with his work in number theory: the Fermat's last theorem, Little theorem of Fermat, Fermat's primes.

No one was able to prove Fermat's last theorem ($x^n + y^n \neq z^n$, when $n > 2$) for 400 years, until Andrew Wiles in 1994.

He also developed algebraic coordinate geometry independently of Descartes.

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Pascal reached out to Fermat to ask whether his reasoning is correct.
Pascal's solution was correct, but much more complicated than the one of Fermat.

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The first letters between Pascal and Fermat are lost. Only 7 of them are preserved. They never met in person, even though they were very close to each other.

Determining the fair value of something is the central question in mathematics!

Fermat and Pascal confirmed this, as their problem turned out to be very improtant. Why?

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He was the first to come up with the term *expected value* (expectation).

The 7 giants of statistics

1. **Jacob Bernoulli (1654 - 1705)**
   Famous Swiss mathematician. The law of large numbers.

2. **Abraham De Moivre (1667 - 1754)**
   French Hugenot, who escaped to England. Friend of Newton. Proved the Central Limit Theorem for binomial distribution.

3. **Pierre Simon Laplace (1749 - 1827)**
   From middle-class French to top scholar in Paris. First proof of a general CLT.

4. **Carl-Friedrich Gauss (1777 - 1855)**
   The prince of mathematics. The method of least squares.

5. **Karl Pearson (1857 - 1936)**
   Great organizer. Founded first statistics department.

6. **Ronald Fisher (1890 - 1962)**
   Multiple contributions transforming mathematical statistics.

7. **Jerzy Neyman (1894 - 1981)**
   Adopted Fisher's framework, brought it to continental Europe.

Take 5000 chocolate bars. 2000 of them are dark, but you don't know this in advance.

How many of them should you sample (check out), to be sure that the observed frequency is within 2% of the true frequency (2=5), with very high probability (say 1=1000)?

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This was the question asked by Jacob Bernoulli. He showed that the observed frequency will get infinitely close to the true frequency, if we take big enough sample (i.e., the Law of Large Numbers).

Stated by Nicolaus Bernoulli (nephew of Jacob and Johann) and proved by his cousin Daniel (the son of Johann).

The paradox

Toss a fair coin until you get an H.

If you toss H the first time, you get 2 coins. If, at the 2nd time - 4 coins, $\cdots$, if at the i-th time, you get 2^i coins.

How much coins you will pay to play the game?

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Abraham de Moivre published a complete solution of the problem in his *The Doctrine of Chances* using calculus.

In fact, de Moivre showed how a collection of random observations would distribute themselves around their average, i.e., he found the Normal distribution.

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properties (e.g., the Fibonacci numbers, the numbers , e.)
What are those unique properties for the normal distribution?

The normal distribution is the only one with these properties!

[1] http://www.cs.toronto.edu/~yuvalf/CLT.pdf