01:640:437 HISTORY OF MATHEMATICS

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W4: The problems of Diophantus. The paradoxes of Zeno.

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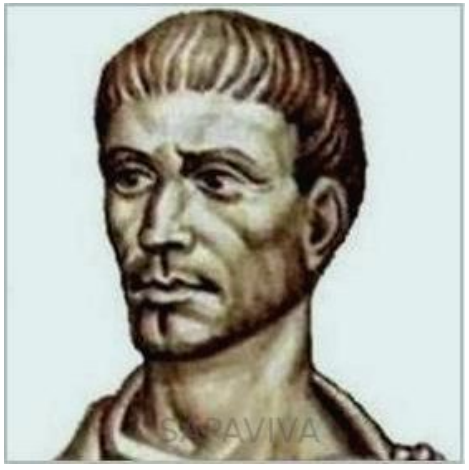
Diophantus (200AD - 280AD)



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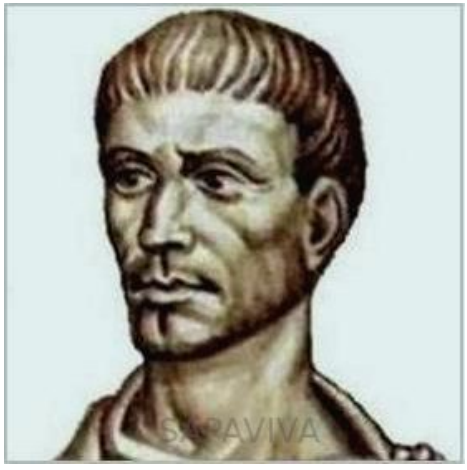

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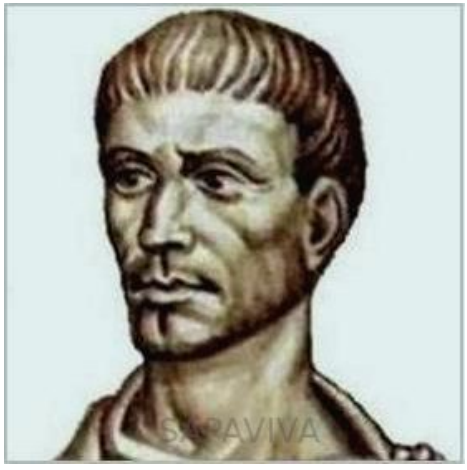

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- Little is know for Diophantus except that he lived in Alexandria.
- He introduced symbolism in mathematics! (in Babylon and Egypt all problems were written with words)
- His symbols were quite different. For instance, $x^3 - 3x^2 + 3x - 1$ was written as:

$$K^Y \alpha \varsigma \gamma \Lambda \Delta^Y \gamma \mathring{M} \alpha$$

Some of his problems

Most of Diophantus' problems were with more unknowns than equations!

1. Find two numbers x and y, such that $x + y = a$ and $x^2 + y^2 = b$. Here a and b are given numbers.

   – a problem solved by the Babylonians.

   – how would you solve it for $a = 20$ and $b = 208$?

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2. Divide a given square number into two squares. For example, find x^2 and y^2, such that $x^2 + y^2 = 16$?

   – Diophantus was almost always satisfied with finding one solution of a problem

   – Here is how he got one for this problem: $16 - x^2$ must be a square, so let $16 - x^2 = (ax - 4)^2$, for some integer a (why did he take -4?).

Some of his problems

3. Find 3 squares with certain ratio between the two differences between the largest and the middle and between the middle and the smallest?

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   Actually, he wants to solve $x^3 + y = (x+y)^3$. Diophantus begins by assuming $x = 2y$. Then, he got $19y^2 = 1$, but here he note that 19 is not a square, so he come back!

   He sees that $19 = 3^3 - 2^3$, so he needs to replace 2 with some z, such that $(z+1)^3 - z^3$ is a square. He uses his previous technique (see Problem 2) to find $z = 7$.

Diophantus - influence

Diophantus' only survived work was highly influential:

– It was studied later by Islamic authors.

– Many of his problems were included in the work "Algebra" by Bombeli in 1572.

– Even Pierre Fermat studied these problems in the 17th century and proved more general results!

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- Eleatics believed in the unity and permanence of being contrasting the Pythagorean ideas of multiplicity and change!

The paradoxes of Zeno

Zeno challenged the Pythagoreans by proposing several paradoxes. We know about 9 of them today...

Most of the paradoxes are equivalent. We will focus on the most famous paradox about Achilles and the tortois..

Zeno's Paradox

Achilles and the tortoise

When Achilles (A) will reach the position where the tortoise (T) started, the tortoise will have moved further ahead.

Other paradoxes and conclusions

Other paradoxes of Zeno are: the Dichotomy paradox, the Arrow paradox, the Stade paradox, etc.

- All of the paradoxes of Zeno are paradoxes of motion.
- These paradoxes have an impact on the development of mathematics even today.
- There are confronting views on whether the paradoxes are resolved or not.
- In these paradoxes, Zeno was the first one to use “proof by contradiction” arguments!